

Homework Answers

Section 1.1 exercises (page 9) (12 problems):

2

4

10

12

14

20

32

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Notes

Functions: Overview

- ▶ What is a function?
- ▶ Domain and Range

Notes

What is a Function?

Definition (Function)

A function is a rule (or formula) that calculates a unique output for every input.

Examples

▶ $f(x) = 3x$

▶ $h(x, y) = 2x + 3y$

▶ $g(a) = a^2 - 4$

▶ $\exp(a, b) = a^b$

The input for a function can be one or more values; the output can be more than one value, but for this course the output will always be one number.

Question

Why would a function have an output (or answer) consisting of multiple pieces?

Notes

Testing if something is a function

Assume that the function takes one input, and produces one output, unless specified otherwise, for this course.
 $y = f(x)$ is a symbolic way to say "y is a function of x".
The number pair $(x, f(x))$ is the pair of numbers consisting of the input and output of the function f .
We also write this number pair as (x, y) .

Definition (Vertical Line Test)

If every vertical line in the xy -plane intersects the graph of the function f in at most one point, then f is a function.

Examples

Which of these is a function, and which is not?

- ▶ $f(x) = \sqrt{x}$
- ▶ $f(x) = \pm\sqrt{x}$
- ▶ $f(x) = 2^x$

Notes

Example of a function

Question

Write a function that calculates the area of a circle, given its radius.

Use the function to find the area of a circle of radius 2in.

Solution

- ▶ $A(r) = \pi r^2$
- ▶ $A(2) = \pi 2^2 = 4\pi \text{ in}$

Question

When doesn't this function make sense?

Notes

Domain

Definition (Domain)

The domain is the set of possible input values.

The domain is either the set of all real numbers - the natural domain - or is a restricted subset of the natural domain.

Examples

- ▶ $f(x) = 3x, x > 4$ (Explicit restriction of the domain)
- ▶ $f(x) = \sqrt{2x - 4}$ (Restriction in context: $2x - 4$ must be non-negative.)
- ▶ $f(x) = \frac{1}{x}$ (Restriction in context: $x \neq 0$)
- ▶ $f(x) = 4x^3 - 3x^2$ (No restrictions)

Notes

Range

Definition

Range The range is the set of all possible output values.

The range is dependent on the domain of the function.

Example

What is the difference between the range of the following two functions?

- ▶ $f(x) = x^2$
- ▶ $f(x) = x^2, x \geq 2$

Notes

Notation

Both the domain and range are sets.

When drawing the set as an interval, the boundary points are the two endpoints.

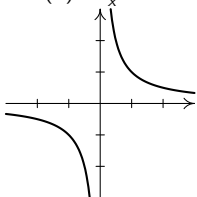
- ▶ $(-\infty, \infty)$ - The set of all real numbers
- ▶ $(-3, 4]$ - The set of all real numbers between -3 and 4, including the right boundary point.
- ▶ $[0, \sqrt{41}]$ - The set of all real numbers between 0 and $\sqrt{41}$, including the boundary points.
- ▶ $(-2, 2)$ - All numbers between -2 and 2, not including the boundary points.
- ▶ $(-\infty, -2] \cup (2, \infty)$ - All numbers except those bigger than -2 but less than or equal to 2.
- ▶ $\{0, 2, 4\}$ - Only the three numbers 0, 2, and 4.
- ▶ $\{2n, n > 0\}$ - All positive even numbers.

Notes

Finding the Domain and Range of a function

Example

- ▶ Find the domain and range of $f(x) = \frac{1}{x}$.



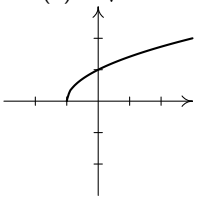
- ▶ You can not divide by 0, so the domain is everything except 0.
- ▶ Domain = $(-\infty, 0) \cup (0, \infty)$
- ▶ The value y takes on every real value
- ▶ Range = $(-\infty, 0) \cup (0, \infty)$

Notes

Finding the Domain and Range of a function

Example

- ▶ Find the domain and range of $f(x) = \sqrt{x+1}$.



- ▶ The value inside the $\sqrt{\quad}$ must be zero or positive. So $x+1 \geq 0$, or $x \geq -1$.
- ▶ Domain = $[-1, \infty)$
- ▶ The value y will always be zero or positive.
- ▶ Range = $[0, \infty)$

Notes

Homework

Section 1.2 exercise (page 19) (12 problems):

- 1
- 2
- 3
- 4

6 Domain =
Range =

8 Domain =
Range =

14 Domain =
Range =

16 Domain =
Range =

Notes

Notes